# Author's  Solution -Dec 2025

**Given:**

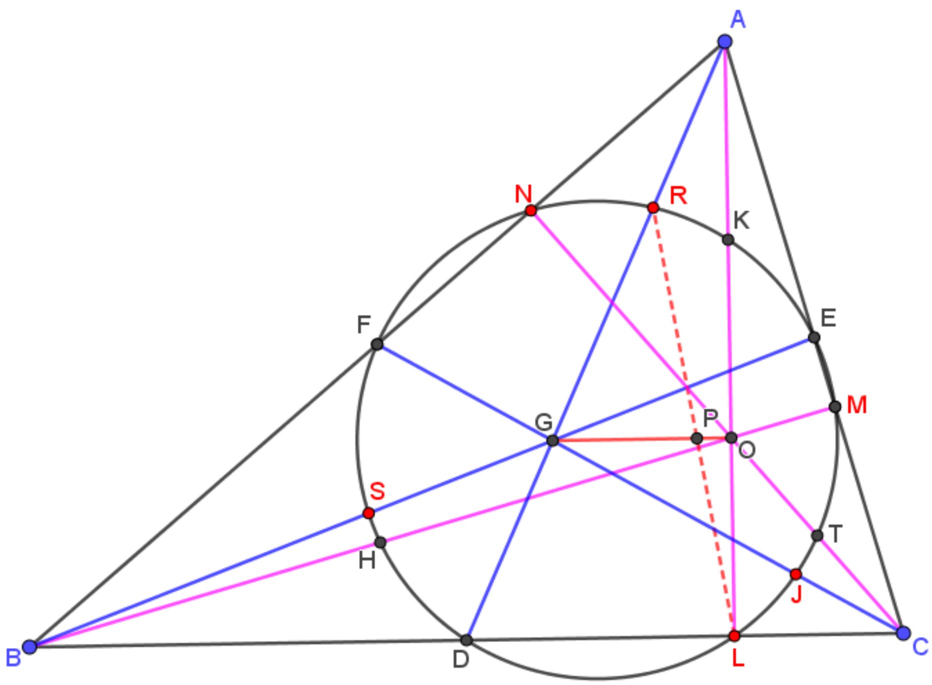
In the picture, in ΔABC, the Twelve Points Circle(TPC) passes through the points D,E & F (midpoints), L,M & N (feet of the altitudes), K, T & H (the second points on the altitudes) and R,S & J (the second points on the medians). G is the Centroid and O is the Orthocentre and GO is the Euler Line.

Prove that the chords RL, SM, NJ & the Euler Line segment GO are concurrent at a point P (say) (And P is the fifth significant point on the Euler line).

**Proof:**

First let us assume that the chords RL, SM & NJ are not concurrent but intersect the line segment GO at different points. Let RL intersect GO at P. Let SM intersect GO at Q and let NJ intersect GO at W. Now, let us prove that $\frac{OP}{PG} = \frac{OQ}{QG} = \frac{OW}{WG}$. If we prove this, then, it will be automatically proved that P, Q & W are one and the same point and that RL, SM & NJ are also concurrent on a point on the line segment GO.

First let us find out $\frac{OP}{PG}$.




RG x GD = SG x GE = JG x GF = $x$ (say) [chords concurrent at Centroid] -------------(1)

Since AL, BM & CN are altitudes, AMLB & ACLN are concyclic.

⟹AO x OL = BO x OM = CO x ON = $y$ (say) [segments of altitudes ] -------------------(2)

For Δ $AGO$, $RPL$ is a transversal. As per Menelaus Theorem

$$\frac{AR}{RG} \times \frac{GP}{PO} \times \frac{OL}{LA} = 1$$

$$\Longrightarrow \frac{AR}{RG} \times \frac{OL}{LA} = \frac{OP}{PG}$$ ---------------------------------- (3)

The Chords DR & LK meets at A outside the circle.

$\Longrightarrow$ AD $\times AR = AL \times AK$

$$AR = \frac{AL \times AK}{AD}$$ ---------------------------------- (4)

(1)$\rightarrow RG = \frac{x}{GD} = \frac{3x}{3GD}$

$\Longrightarrow RG = \frac{3x}{AD}$ ------------------------------------(5) $(\because AD = 3GD \text{ since } AD \text{ is Median } \& \ G \text{ is Centroid})$

(4) ÷ (5) →

$$\frac{AR}{RG} = \frac{AL \times AK}{3x}$$

$= \frac{AL \times 2 \times AK}{2 \times 3x} = \frac{AL \times AO}{6x}$ $(\because AK = KO \text{ since the TPC passes through the midpoint of } AO)$

ie $\frac{AR}{RG} = \frac{AL \times AO}{6x}$ ----------------------------------------(6)

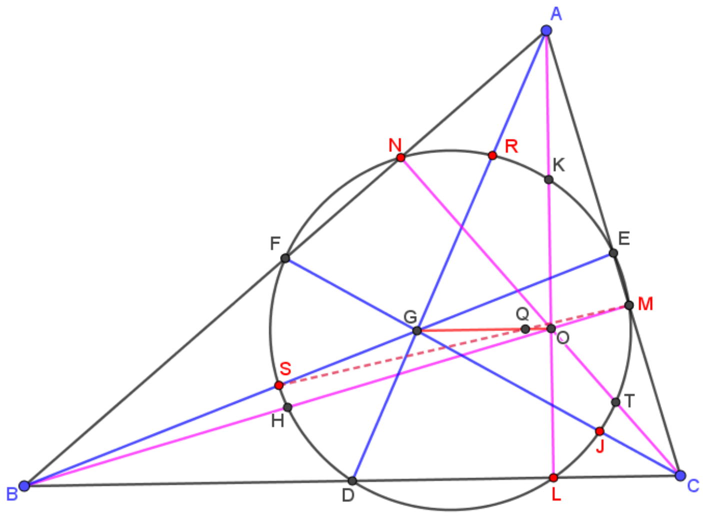
$\Longrightarrow \frac{AR}{RG} \times \frac{OL}{LA} = \frac{AL \times AO}{6x} \times \frac{OL}{LA} = \frac{AO \times OL}{6x}$ ----------(7)

(7) $\rightarrow \frac{AR}{RG} \times \frac{OL}{LA} = \frac{1}{6}\left(\frac{y}{x}\right)$ --------------------------(8) (since AOxOL = y as per 7 above)

(3) & (8) →

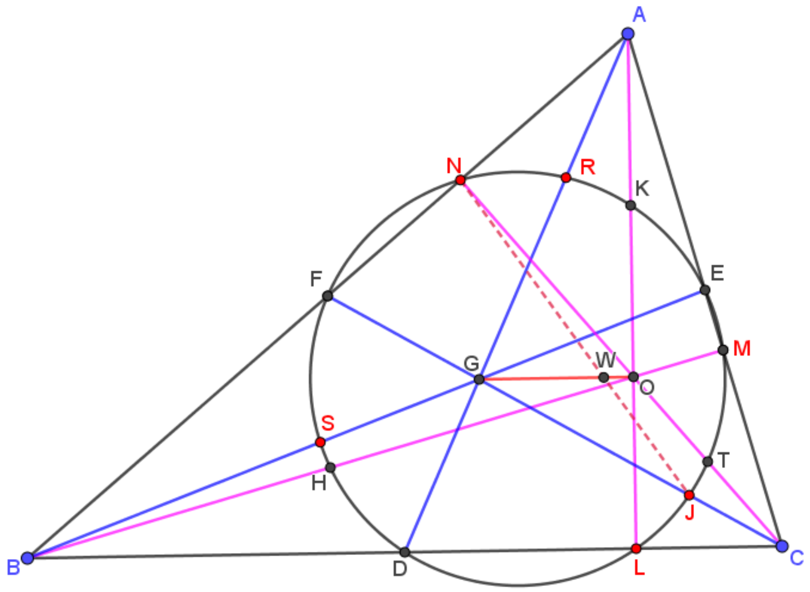
$$\frac{OP}{PG} = \frac{1}{6}\left[\frac{y}{x}\right]$$

SM intersect GO at Q. Similarly, we can prove $\frac{OQ}{QG}$.




$\therefore \frac{OQ}{QG} = \frac{1}{6}\left[\frac{OM \times OB}{SG \times GE}\right] = \frac{1}{6}\left(\frac{y}{x}\right)$ $(\text{we can take } \Delta BGO \ \& \text{ its tranversal } SQM \text{ and prove})$

NJ intersect GO at W. Similarly, we can prove $\frac{OW}{WG}$.

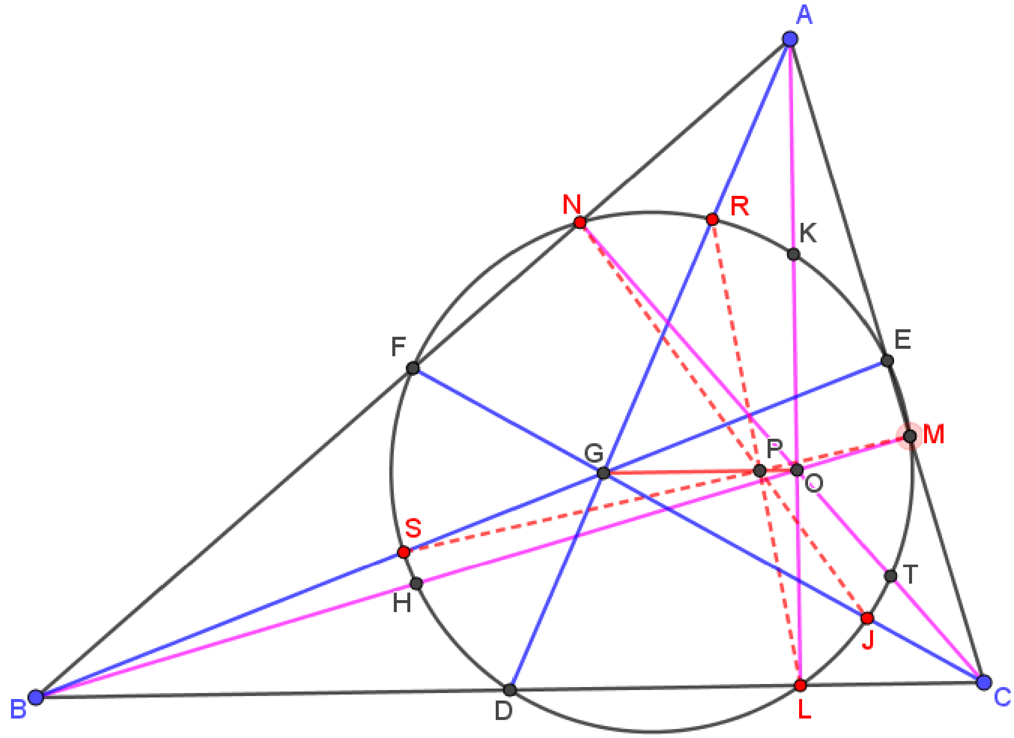



$\therefore \frac{OW}{WG} = \frac{1}{6}\left(\frac{y}{x}\right)$ [we can take $\Delta CGO$ $and\ its\ tranversal\ JWN\ and\ prove$]

$\Longrightarrow \frac{OP}{PG} = \frac{OQ}{QG} = \frac{OW}{WG} = \frac{1}{6}\left(\frac{y}{x}\right)$

$\Longrightarrow P, Q\ \&\ W$ are one and the same point.

Hence, it is proved that the chords RL, SM & JN are concurrent on a point on GO, say P and **P is the 5th significant point on the Euler line**.

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